Controllability of the Korteweg-de Vries equation on a star-shaped network

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Abstract

Control theory has an important place in different scientific disciplines.This allows the study of certain properties of mathematical models that describe physical phenomena. A large part of these models use different types of partial differential equations being of interest to us systems of coupled equations from an applied perspective.

In this talk the Korteweg-de Vries equation will be presented in a star-shaped network. This system is formed by N Korteweg-de Vries equations coupled by the boundary conditions. In the literature (see[1]), there is a recent result proving the exact controllability of this system by using $(N+1)$ controls where N controls act on the external nodes of the network and the other one is a central control. It will be shown that the system is exactly controllable with fewer controls. The system is described by the following equations:

$$\left\{\begin{array}{ll} (\partial_t u_j + \partial_x u_j + u_j \partial_x u_j + \partial_x^3 u_j)(t,x) = 0, & \forall x \in (0,l_j), \forall t > 0, j = 1,..N, \\ u_j(t,0) = u_k(t,0), & \forall t > 0, j,k = 1,..N, \\ \sum_{j=1}^N \partial_x^2 u_j(t,0) = -\alpha u_1(t,0) - \frac{N}{3}(u_1(t,0))^2 + g(t), & \forall t > 0, j = 1,..N, \\ u_j(t,l_j) = 0, & \forall t > 0, j = 1,..N, \\ \partial_x u_j(t,l_j) = g_j(t), & \forall t > 0, j = 1,..N, \\ u_j(0,x) = u_j^0(x), & \forall x \in (0,l_j), j = 1,...,N. \end{array}\right. \tag{1}$$

We use the duality and multiplier method to study the controllability of the linearized system around the origin and the fixed point theory to include nonlinearities.

References

[1] K. Ammari, E. Crépeau. Feedback stablilization and boundary controllability of the Korteweg-de Vries equation on a star-shaped network. SIAM J. Control Optim (2018). 1620 - 1639.