Inverse problems for first-order transport equations by Carleman estimates: global Lipschitz stability with data on hypersurface

Masahiro YAMAMOTO

The University of Tokyo,
Honorary Member of Academy of Romanian Scientists
myama@ms.u-tokyo.ac.jp

Abstract

We consider the transport equation of the first order in a bounded domain $\Omega \subset \mathbf{R}^d$ with smooth boundary $\partial\Omega$:

$$\partial_t u(x,t) + H(x) \cdot \nabla u + p(x)u = F, \quad 0 < t < T,\, x \in \Omega.$$

Here let $H \in C^1(\overline{\Omega})$, $p \in L^\infty(\Omega)$ be given. We fix a hyperplane $\Gamma \subset \overline{\Omega}$ (not necessarily $\widetilde{\Gamma} \subset \partial\Omega$). We consider the following two inverse problems:

Obervability: Let $F = 0$. Determine $u(\cdot,0)$ in Ω by $u|_{\Gamma\times(0,T)}$.

Inverse source problem: Let $R(x,t)$ be given. Determine $f(x)$ by

$$u|_{\Gamma\times(0,T)}, \quad u(\cdot,0)|_\Omega$$

in $\partial_t u(x,t) + H(x) \cdot \nabla u + p(x)u = R(x,t)f(x)$.

In [1], with $\Gamma \subset \partial\Omega$, assuming a generous condition on $H(x)$, we prove a local stability estimate of Hölder type, which holds only in a small subdomain. Here we mainly discuss the stability over the whole domain Ω. It is easily understood that we cannot expect even the uniqueness with $\Gamma = \partial\Omega$ if the stream generated by $H(x)$ rotates.

We establish the Lipschitz stability over Ω in the above two inverse problems, if we choose extra hypersurface $\widetilde{\Gamma} \subset\subset \Omega$ and we use data of u on $\Gamma := \partial\Omega \cup \widetilde{\Gamma}$ over a sufficiently long time interval. We can describe an algorithm how to find such $\widetilde{\Gamma}$ and we can geometrically understand that $\widetilde{\Gamma}$ should cut all the circulating cycles of streams generated by $H(x)$.

The key is a Carleman estimate with piecewise continuous quadratic weight function in space. As for similar works, see [2], [3] and [4] where Carleman estimates are used.

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References

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