Inverse source problems for time-fractional diffusion(-wave) equations

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Abstract

Within the last decade, evolution equations with fractional derivatives have gathered considerable attention among both theoretical and applied disciplines due to their feasibility in modeling physical processes such as anomalous diffusion. In this talk, we consider initial-boundary value problems for time-fractional diffusion(-wave) equations represented by

$$(\partial_t^\alpha - \triangle)u(x,t) = F(x,t), \quad x \in \Omega,\ 0 < t < T,$$

where ∂_t^α ($0 < \alpha \le 2$) denotes the Caputo derivative. We investigate the following three inverse source problems on determining some components in the inhomogeneous term F.

1. Provided that $F(x,t) = g(x)\rho(t)$ and ρ is known, determine the spatial component g by the partial interior observation of u in $\omega \times (0,T)$ ($\omega \subset \Omega$).
2. Provided that $F(x,t) = g(x)\rho(t)$ and g is known, determine the temporal component ρ by the single point observation of u at $\{x^0\} \times (0,T)$ ($x^0 \in \Omega$).
3. Provided that $F(x,t) = g(x-\gamma(t))$ and g is known, determine the orbit γ by the multiple point observation of u at $\{x^j\}_{j=1}^N \times (0,T)$ ($\{x^j\}_{j=1}^N \subset \Omega$).

The starting point for all problems is a fractional Duhamel's principle, which represents the solution in form of a convolution and reduces the problem to the discussion of its homogeneous counterpart. For Problem 1, we prove the uniqueness by utilizing a newly established weak unique continuation property. For Problem 2, the uniqueness and the stability of multiple logarithmic type follow from the applications of a strong maximum principle and a reverse convolution inequality, respectively. For Problem 3, we derive a Lipschitz stability estimate in the case of a localized moving source with the minimum possible observation points, and the uniqueness for the general case is verified with more observations.